Mentoring Advanced Placement
Physics

Topic: Forms of solar energy, mass loss

Quick Question:

In books or songs you may have heard that "we are all made of stars." This is based on the model of stellar evolution where the universe began as a proton cloud. Stars, beginning with mostly hydrogen and a bit of helium, create heavier and heavier elements through fusion steps near the end of their stellar life cycle.

This type of fusion reaction, birthing carbon and most of the other elements we are made of, releases energy only up to iron. Fusing additional protons to iron requires a net energy input.

Q: If this is so, where do elements heavier than iron come from?

Challenge:

1.) As a rough average, 1,340W/m^2 of solar energy, in the form of radiation, is incident on the earth. The atmosphere absorbs some of this energy, some is reflected back, and the earth absorbs the rest before emitting it in some other form.

If 1,340W/m^2 of power reaches the earth, what is the approximate radiation power output of the sun? (Hint: The radius of the earth's orbit is 1.5x10^{11}m.)

2.) In the summed solar fusion reaction $4\ {}^{1}H^{+} + 2e^{-} \rightarrow {}^{4}He^{2+}$, what is the mass loss when going from products to reactants? How much energy does this correspond to?

3.) Using the relationship of energy to mass, roughly how much mass does the sun lose per second? For water, density 1x10^{3}kg/m^3, what volume would this correspond to?

4.) The sun can only burn 10% its initial Hydrogen before core conditions are unsuitable for main phase Hydrogen fusion.

If our sun began with 1.5x10^{30}kg of H, and has consumed roughly 5% already, how many years of H burning are left?

QQ Solution:

The energy input required is thought to come from supernovas and other cataclysmic astronomic events. When fusion events stop, old stars rapidly collapse.

The massive amount of energy involved in this collision drives formation of heavy elements. Then, after explosions spew out the heavy materials, spontaneous decay continues alterations, and the resultant array of dust is incorporated into forming stars or other astral bodies.

Challenge Solutions:

1.) The radius of the earth's orbit is $1.5x10^{11}$m. Assuming that solar radiation is spherically symmetric, all points at this radius from the sun will experience the same 1340W/m^2. Since solar radiation is in flux radially outward, the sum power moving through an imagined spherical shell will be equal for all shells centered on the sun with radius greater than that of the sun.

$(4\pi(1.5x10^{11}m)^2)(1340W/m^2) = 3.8x10^{26}W$ = approximate radiative power output of the sun.

2.) 4 ^{1}H have a mass of 4.04amu, ^{4}He has a mass of 4.00amu. 1amu = $1.66x10^{-27}$kg.

$4.04amu - 4.00amu = 0.04amu = 6.64x10^{-29}kg$

Using Einstein's ever famous equation,

$mc^2 = (6.64x10^{-29}kg)(3x10^8m/s)^2 = 5.97x10^{-12}J$

3.) $(3.8x10^{26}W)(1s) = 3.8x10^{26}J$, $3.8x10^{26}J/ (3x10^8m/s)^2 = 4.2x10^9kg$ lost per second.

For water, that would be $4.2x10^9kg / 1x10^3kg/m^3 = 4.2x10^6m^3$

An Olympic pool is $2,500m^3$ →

$4.2x10^6m^3 / 2,500m^3 = 1680$ Olympic size swimming pools of mass lost every second.

4.) The mass of the sun is gauged to be 71% Hydrogen. At a total $1.989x10^{30}$kg this is $1.41x10^{30}$kg of Hydrogen. Since roughly one half of the usable hydrogen has been burned, (5%), the current amount is 95% of the initial. 5% of the initial mass, the amount remaining still available, is 7.5x1028kg.

Of 4.03amu only 0.03amu, or ~0.7% of the mass is converted to energy, so annihilating $4.2x10^9$kg requires $(1/0.007)(4.2x10^9kg) = 6x10^{11}kg$ of Hydrogen.

$7.5x10^{28}kg / 6x10^{11}kg/s = 1.25x10^{17}s = 3.96x10^9yrs$ or ~4 billion years.

This fits well with the estimate that the sun has been burning for 4.5billion years and will burn in one form or another for a total 10 billion years.