Mentoring Advanced Placement
Physics/Mathematics

Topic: Modeling heat flow

Quick Question:

- The meat counter at your local grocery store sells cuts of meat with bone in, and cuts already de-boned.
- You notice this having just watched Braveheart, and wanting to celebrate the Scottish by eating mutton.
- Your friend feels the same way, so you both buy two pound roasts. Yours bone in, theirs de-boned.

Q: With identical ovens, cooking at the same temperature, whose meat cooks faster? What explains this?

Challange:

- In 1985 an Australian steel company wanted to study the feasibility of pouring molten steel onto a cooled rotating copper drum to produce continuous steel sheets of considerable length.
- They preferred mathematical modeling to building a potentially useless machine, and contacted a mathematics in industry workshop to create a rough model for judgment. Here is the problem as they figured it.

The drum surface speed will be roughly 1m/s. Desired sheet thickness is between 1mm and 10mm, and thickness can be controlled by how quickly molten steel is released onto the drum. The steel closer to the drum cools first, while the upper part remains liquid and forms a puddle. This puddle will eventually cool and solidify, but if the drum turns too quickly the puddle will drip and pour off, so the sheet must be solid before the drum turns too far.

1.) Write a basic equation using variables to describe how long the puddle will be?

Look at a one dimensional cross section, (a vertical segment with the barrel at the bottom). For simplification, treat heat flow as a constant.

The steel will be at 1,400°C. To solidify, it will have to drop in energy by the amount of its latent heat. The latent heat of fusion for steel, the energy required to undergo phase change from solid to liquid, is 2.7×10^5 J/kg.

2.) With the exceedingly rough approximation that our one dimensional sample has a mass of 1g, and that heat energy flows off into the drum at a rate of 30J/s, what is the time required for the full thickness to solidify for a 10mm sheet?

3.) What is the puddle length for this scenario?

4.) How big would the drum have to be for this segment to be fully solid before the drum rotates15 degrees about its axis? (To avoid dripping.)

5.) Does this seem feasible?

QQ Solution:

The roast with bone in cooks faster. Since the bone contains less water and has a lower specific heat than the meat, it heats more rapidly and conducts heat from the oven into the center of the roast. This allows the cut to cook from both the inside and the outside. Irregularities in the cut's shape, and bone location, will likely result in some pockets that cook more thoroughly than others, but the net affect should be edible before the solid roast.

Challenge Solution and Discussion:

1.) For drum surface speed V, the puddle length (l) will be $l = Vt_h$, where t_h is the time required for the sheet to solidify to its full thickness h.

So, to find the puddle's length, and how big a drum will prevent dripping, we must find t_h. This departs into modeling of one dimensional heat transfer from solid steel to the drum, and across the moving boundary between the liquid and solid steel. It can be assumed that heat flow from the liquid steel to the air is negligible.

Calculating heat flux for specific cases is complicated. Heat flow is certainly not constant, but to provide a simpler modeling problem we will imagine that it is.

2.) For the assumed case, $(2.7\text{x}10^5 \text{J/kg})(1\text{x}10^{-3}\text{kg})/ (30\text{J/s}) = 9\text{s} = t_h$

3.) With drum speed of 1m/s, (9s)(1m/s) = 9m

4.) $2\pi r * (15^\circ/360^\circ) = 9\text{m} \rightarrow r = 34.4\text{m}$

5.) This is not very feasible. A water filled rotating copper drum with a diameter of 69m would be an enormously expensive feat of construction. In real life, the group modeling this problem found roughly the same result and saved the steel company from wasting money on an unworkable physical model.

Now, feel free to depart into a more realistic discussion of the modeling process.

- Show how a one dimensional model involves interface between the drum, the solid layer, the liquid layer, and a boundary at the air.
- Show how variables can be named to describe the location, taking the drum surface to be $x=0$, and the liquid solid interface to vary as a function of time.
- Discuss how functions describing the temperature at each location will vary due to the properties of liquid steel (assumed to have uniform temp.), solid steel (a curved gradient), and the copper drum (assumed to be at constant temperature/ a heat sink/ modeled as semi infinite, $-\infty<x<0$).
- Show that the temperature curves, and heat flows, must be constructed as continuous functions across the three regions.
- Discuss how thermo dynamic equations describe the flow rate, and how applying specific temperatures of the drum, at 150C, the steel at 1400C, the properties of copper and steel for thermal conductivity, thermal diffusivity, latent heat energy, and height of the piece, can all come in to take a general solution and provide an answer for a specific occasion.
- If time and curiosity permit, discuss the use of Boltzmann transformations and solving ODE's.